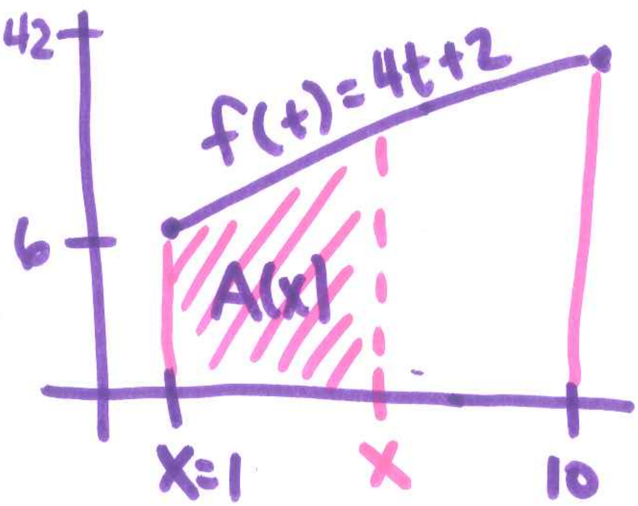
# Chapter 10 - Day 2

Ex: find a formula for A(x)

$$A(x) = \int_1^x (4t+2)\,dt$$

(we're finding the definite integral from [1, x]




then A(x) is the area of a trapezoid

$$\frac{b\cdot(h_1+h_2)}{2}$$

$$A(x) = \frac{(x-1)(f(1)+f(x))}{2} = \frac{(x-1)(6+4x+2)}{2}$$

$$= \frac{(x-1)(4x+8)}{2} = \frac{(x-1)\,\overset{2}{\cancel{4}}(x+2)}{\cancel{2}}$$

$$= 2(x-1)(x+2) = 2(x^2+x-2)$$

$$= 2x^2+2x-4$$

We found $A(x) = 2x^2 + 2x - 4$

Notice that $A'(x) = 4x + 2$

Therefore $A(1) = \int_1^1 (4t+2)\,dt = 0$

and $A'(x) = \frac{d}{dx}\Big(\underbrace{\int_1^x (4t+2)\,dt}_{A(x)}\Big) = 4x+2$

More generally,

$$A(a) = \int_a^a f(t)\,dt = 0$$

$$A'(x) = \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$$

if $F(x)$ is an antiderivative of $f(x)$,

So $F'(x) = f(x) = A'(x)$

By our constant function theorem, $c$ exists such that $F(x) = A(x) + c$

In fact,
$$\int_a^b f(t)\,dt = A(b)$$
$$= A(b) - 0$$
$$= A(b) - A(a)$$
$$= [A(b)+c] - [A(a)+c]$$
$$= F(b) - F(a)$$

# The Fundamental Theorem of Calculus

**Part 1:** Let $f(x)$ be a continuous function on $[a, b]$, then $A(x)$, defined as $A(x) = \int_a^x f(t)\,dt$ is an antiderivative of $f(x)$. Thus

$$A'(x) = \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$$

**Part 2:** Let $F(x)$ be any antiderivative of $f(x)$ so that $F'(x) = f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

**Notation:** $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

# Properties of Definite Integrals

① $\int_a^a f(x)\,dx = 0$

② $\int_a^b k\,f(x)\,dx = K\int_a^b f(x)\,dx$

③ $\int_a^b (f(x) \pm g(x)) = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

④ $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

⑤ $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

⑥ if $m \le f(x) \le M$ on $[a,b]$ then

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$